# Variance Associates: Part A[1](#_bookmark0)

Variance Associates (VA) is a commercial aircraft leasing firm. It has two customers, FearUs and Oops, both of which are major overnight package delivery companies. Airlines and overnight package companies normally own only a portion of their total fleet of aircraft, and lease the balance on an as-needed basis. FearUs and Oops have long-term contracts with Variance, under which they have the option to request planes from VA on a monthly basis. Specifically, each month FearUs can request up to 4 planes for the succeeding month, at a price of $150,000 per month for each plane requested. Oops can also request up to 4 planes for the succeeding month, at a price of $150,000/plane per month.

Variance owns 8 planes, in order to cover its contracts. It has had both good and bad years in the past, but cumulatively it is just breaking even. Out of frustration, the president of Variance Associates hires a consultant.

McBain & Co. assigns Poindexter Harbus (Fecbus ’18) to assess the problem and deliver a solution. The president of VA explains to Poindexter that although VA has great margins on its leased planes, the firm’s total performance has been uneven. Poindexter learns that VA has *fixed* costs of $75,000/plane per month (i.e., whether or not the plane was leased that month), but that all other costs incurred when a plane is in service (e.g., fuel, maintenance, crews) are covered by the customer at the customer’s expense.

Poindexter begins his analysis by examining Variance’s data on the demand for planes by each customer. He computes relative frequencies and tabulates the following probabilities:

FearUs Demand for Aircrafts Oops Demand for Aircrafts

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| demand | 0 | 1 | 2 | 3 | 4 | demand | 0 | 1 | 2 | 3 | 4 |
| probability | 0.2 | 0.1 | 0.4 | 0.1 | 0.2 | probability | 0.1 | 0.2 | 0.4 | 0.2 | 0.1 |

After appraising the situation, Poindexter reports to the president of Variance Associates:

* VA should expect 4 of their planes to be on the ground at VA each month.
* Under VA’s current business policies, the expected cash flow (revenues less expenditures) each month are in fact $0.

1Based on Reiss and White (1997)

**Question 1:** How did Poindexter arrive at these conclusions? (i.e., verify Poindexter’s expected value numbers.) What is the expected demand from FearUs? From Oops?

# A New Strategy...

In order to improve Variance’s profitability, Poindexter proposes a new, two-tier strategy for VA:

* Sell off one plane, and keep only 7 to meet demand.
* To satisfy the customers, offer a new policy: If a customer ever asks for a plane and VA does not have one, VA will *pay the customer* $150,000. (Actually, VA offers a “get your next plane for free” policy rather than paying the customer directly. But that is more complicated to analyze so let’s assume VA pays the customer directly).[2](#_bookmark1)

To determine expected profits under the new strategy, Poindexter needs to determine the probability that total demand would be 8 planes (i.e., a “stock out” event). To do this, Poindexter multiplies the probability distributions for each customer’s demand to construct a probability table:

## FearUs Demand

**Oops**

0 1 2 3 4

0

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.02 | 0.01 | 0.04 | 0.01 | 0.02 |
| 0.04 | 0.02 | 0.08 | 0.02 | 0.04 |
| 0.08 | 0.04 | 0.16 | 0.04 | 0.08 |
| 0.04 | 0.02 | 0.08 | 0.02 | 0.04 |
| 0.02 | 0.01 | 0.04 | 0.01 | 0.02 |

1

0.1

0.2

## Demand 2

3

4

0.2 0.1 0.4 0.1 0.2

0.4

0.2

0.1

From this he calculates a probability distribution for total demand:

Probability Distribution of Total Demand

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| total demand | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| probability | 0.02 | 0.05 | 0.14 | 0.17 | 0.24 | 0.17 | 0.14 | 0.05 | 0.02 |

2Some of you schemers may happen on a clever strategy to defraud Variance here. To head you off at the pass, assume that Variance is sharp enough not to let FearUs know how many planes Oops has requested for the coming month, and vice versa, and that there is enough competition between FearUs and Oops that they’re not about to tell each other, either.

which he uses with the revenue, fixed cost, and compensation policy cost numbers to compute expected profits under the new strategy.

**Question 2:** Briefly explain how Poindexter derived the distribution for total demand. Specifically, why is P(total demand = 1) = *.*05? (Hint: Start from his 5 *×* 5 table)

**Question 3:** Under Poindexter’s new strategy, what does Poindexter’s analysis indicate VA’s expected profits will be?

The president of VA likes Poindexter’s results. But after pondering Poindexter’s reasoning for a few minutes, he wonders: “Why downsize by just one plane if we expect demand to be only *four* planes?” He reasons, “If that’s what we expect, we should just carry four planes. Instead of carrying enough planes to handle the *maximum* demand, we’ll carry enough to handle the *average* demand, and suitably compensate FearUs and Oops with a free plane if we’re short.”

**Question 4:** Will VA see higher expected profits with only 4 planes to meet customer demand? Why or why not? (Hint: Make another table.)

**Question 5:** Now that you have the results for the expected number of planes, what is the optimal number of planes for VA to own?

# Variance Associates: Part B The Analysis Continues...

Variance Associates’ accountant, Flim Flambert, now joins the fray. Flim is concerned with the *risk* in VA’s monthly cash flow position under the old and new strategies. To get a measure of this risk, Flim computes the *standard deviation* in monthly income. He then points out a problem in Poindexter’s analysis, and announces “I can’t reconcile Poindexter’s probabilities with our past cash flows. We have more variability in our cash flows each month than Poindexter’s figures imply.”

Flim justifies his conclusion with the actual relative frequencies of historical monthly cash flow amounts for VA:

Probability Distribution of Historical Monthly Cash Flow (in $ 000s)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| cash flow | ($600) | ($450) | ($300) | ($150) | $0 | $150 | $300 | $450 | $600 |
| probability | 0.08 | 0.09 | 0.08 | 0.10 | 0.23 | 0.14 | 0.19 | 0.03 | 0.06 |

Using the above table, Flim calculates the standard deviation in monthly cash flow under the old policy of keeping 8 planes.

**Question 6:** What are the variance and standard deviation in monthly cash flow (under VA’s old policy), based on Flim’s cash flow probabilities? (**Warning:** If you use the Excel commands var() or stdev() functions to do this, you will get the wrong answer. Why?)

**Question 7:** What is the variance and standard deviation in monthly cash flow (under VA’s old policy), using Poindexter’s probability distribution for total demand? (Hint: Make another table.)

In order to reconcile the discrepancy between their variance estimates, Flim and Poindexter decide to go back to square one. Using Variance’s data on the number of planes demanded by each customer each month, Flim tabulates the relative frequencies of all the **joint events** and obtains the following probability table:

## FearUs Demand

**Oops**

0 1 2 3 4

0

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.08 | 0.02 | 0 | 0 | 0 |
| 0.07 | 0.05 | 0.07 | 0 | 0.01 |
| 0.03 | 0.02 | 0.21 | 0.03 | 0.11 |
| 0.01 | 0.01 | 0.10 | 0.06 | 0.02 |
| 0.01 | 0 | 0.02 | 0.01 | 0.06 |

1

0.1

0.2

## Demand 2

3

4

0.2 0.1 0.4 0.1 0.2

0.4

0.2

0.1

**Question 8:** What can we infer from the table about the demand patterns of the two customers?

**Question 9:** In their analyses, Poindexter and Flim used different probability tables for the joint distribution of FearUs’ and Oops’ demands. In doing so, they arrived at the *same number* for the *expected value* of total profit under the 8 plane policy. They did *not* arrive at the same number for the *standard deviation* of total profit, however. Why not?

With the new probability table, Flim and Poindexter re-analyze Poindexter’s original proposal to improve VA’s profitability. Specifically, using Flim’s new probability table they calculate the optimal number of planes for VA to carry, and the corresponding expected profit.

**Question 10:** What is the optimal number of planes to own, if VA compensates customers with

$150k/plane for shortages? What are VA’s expected profits under this strategy?

# Summary Questions:

1. What is the optimal level of capacity for a firm facing uncertain demand: enough to cover maximum demand, average demand, or something else? Why—what are we trading off here? What information do we need to answer this for a particular firm?
2. If a firm has fixed costs and capacity constraints, which is better, to have customers whose demands positively covary or whose demands negatively covary? Why? What does this imply about the firm’s capacity requirements and, given that, capital utilization rates?