Central University of Technology, Free State

MAIN ASSIGNMENT
MATHEMATICS 2
WISS22AI/MAT12A/MAT126A

INSTRUCTIONS TO CANDIDATES AFTER THEY DOWNLOADED THIS FILE

1. **READ THE DECLARATION AT THE BOTTOM OF THIS PAGE.**
2. **NUMBER THE PAGES OF YOUR WRITTEN WORK.**
3. **PUT YOUR SURNAME, INITIALS, STUDENTNUMBER AND SIGNATURE ON EACH PAGE.**
4. **WRITE YOUR SOLUTIONS IN THE SAME ORDER THAT THEY APPEAR IN THIS PAPER.**
5. **YOU HAVE UNTIL 09h00, 20 JANUARY 2021 TO FINISH THIS PAPER.**
6. **COMPILE AND UPLOAD, IN ONE PDF FILE, YOUR WRITTEN SOLUTIONS OF SECTION A ONLY (ANY OTHER FORMATS WILL NOT BE MARKED).**
7. **ANSWERS TO SECTION B SHOULD BE FILLED IN ON eTHUTO.**

SUBJECT:	MATHEMATICS 2
SUBJECT CODE:	WISS22AI/MAT12A/MAT126A
EXAMINER:	DR HE BRINK
MODERATOR:	DR M ERASMUS
DATE OF ASSESSMENT:	19-20 JANUARY 2021
SESSION:	09H00
DURATION:	
CALCULATOR:	NON-PROGRAMMABLE
TOTAL:	100
DECLARATION:	By submitting my work on eThuto, I declare that: • I will complete this assignment/assessment in accordance with CUT's Code of Academic Integrity • all the work included in this assignment/assessment will be my own work; • I will not consult any person or means other than that allowed in the assignment/assessment instructions.

SECTION A [50]

1. MECHANICAL AND CIVIL ENGINEERING

The velocity v of an object is described by

$$v = 0{,}23\sin t + \cos 3t$$

Use the Newton Raphson iteration formula to calculate t (correct up to three decimal places) when $v = 0$. Start with $t = 0{,}5\,s$.

1. ELECTRICAL ENGINEERING [6]

An a.c system is represented by

$$i = 0{,}23\sin t + \cos 3t$$

where i is the current running through the system. Use the Newton Raphson iteration formula to calculate t (correct up to three decimal places) when $i = 0$. Start with $t = 0{,}5\,s$.

2. MECHANICAL AND CIVIL ENGINEERING

A particle moves in a plane according to

$$x = (t+1)e^{-t};\quad y = 2te^{-t}$$

Determine the simplified expression for the gradient

$$m = \frac{dy}{dx}$$

of the tangent to this path.

2. ELECTRICAL ENGINEERING [5]

In an electric circuit, the resistance R, voltage v, current i, inductance L and capacitance C, are related by the equation

$$R = \frac{L}{C}\frac{di}{dv}$$

If it is given that

$$v = (t+1)e^{-t};\quad i = 2te^{-t},$$

determine the simplified expression for $\frac{di}{dv}$.

3. MECHANICAL AND CIVIL ENGINEERING

Length and width of a rectangle are measured with errors of $\pm 3\%$ and $\pm 5\%$ respectively. Estimate the maximum percentage error in the area.

3. ELECTRICAL ENGINEERING [6]

Two resistors in an electric circuit are measured with errors of $\pm 3\%$ and $\pm 5\%$ respectively. Estimate the maximum percentage error in the product of the resistors.

4. MECHANICAL AND CIVIL ENGINEERING

The displacement x of a moving object at time t is given by the equation

$$x = \cos^2 t \sin^3 t$$

The average displacement over the interval $[0;\pi]$ is given by

$$\bar{x} = \frac{1}{\pi}\int_0^{\pi} x\,dt$$

Calculate the value of $\bar{x}$.

4. ELECTRICAL ENGINEERING [7]

The voltage v and current i across a pure capacitance is given by

$$v = \cos^2 t \text{ and } i = \sin^3 t$$

The average power P over the interval $[0;\pi]$ is given by

$$P = \frac{1}{\pi}\int_0^{\pi} vi\,dt$$

Calculate the value of P.

5. MECHANICAL ENGINEERING

An object of mass m falls from rest and its velocity v given by

$$m\frac{dv}{dt} = mg - kv \quad v(0) = 0$$

where k is a positive constant and g is gravitational acceleration. Solve the above differential equation by making use of the integrating factor method.

5. CIVIL ENGINEERING

A tank full of water is being drained out through an opening. The rate of change of the height H of the water in the tank is given by the differential equation

$$\frac{dH}{dt} + H = e^{-t}, \quad H(0) = 10$$

Solve the above differential equation by making use of the integrating factor method.

5. ELECTRICAL ENGINEERING [7]

A circuit consists of a resistor of resistance R, and a capacitor of capacitance C, in series, and is described by the differential equation

$$RC\frac{dV}{dt} + V = E, \quad V(0) = 0$$

where E is the constant applied e.m.f. and V is the voltage across the capacitor. Solve the above differential equation by making use of the integrating factor method.

6. MECHANICAL AND CIVIL ENGINEERING

The volume V (in m^3) and pressure P (in kPa) of a gas in a cylinder is measured in an experiment and the results are shown in the table below:

V	0,1	0,3	0,5	0,7	0,9
P	9,01	8,23	8	5,90	4,03

Make use of Simpson's rule to calculate the work done W by the gas on the face of the piston.

6. ELECTRICAL ENGINEERING [6]

A voltage v (in V) has the values at time (in s) intervals as shown in the table below:

t	0,1	0,3	0,5	0,7	0,9
v	9,01	8,23	8	5,90	4,03

Make use of Simpson's rule to calculate v_{RMS^2}.

7. Use the first three non-zero terms of the Maclaurin series of $\frac{1}{(1-t)^3}$ to calculate [7]

$$\left(\frac{3}{4}\right)^{-3}$$

8. Solve the following system by making use of Gauss elimination: [6]

$$\begin{aligned} 5x + 5y + 5z &= 7 \\ x + 2y + 4z &= 2,4 \\ 4x + 2y &= 4 \end{aligned}$$

SECTION B IS ON THE NEXT PAGE

SECTION B [50]

1. $\frac{\partial}{\partial v}\left(\tan^{-1}(vx)\right) =$ [2]

2. $\frac{d}{dt}\left(e^{\sin^{-1} t}\right) =$ [2]

For questions 3-5 consider the following set of parametric equations:

$$x = 6\tanh\theta; \quad y = 3\operatorname{sech}\theta$$

3. Consider the statements: [2]

(i) $\frac{dx}{d\theta} = 6\operatorname{sech}^2\theta$ (ii) $\frac{dy}{d\theta} = 3\operatorname{sech}\theta\tanh\theta$

Which is/are correct/incorrect?

4. $\frac{dy}{dx} =$ [2]

5. $\frac{d^2y}{dx^2} =$ [3]

For questions 6-7 consider

$$z = r^3\theta + e^{r+\theta^2} + \theta\sin r$$

6. $\frac{\partial z}{\partial \theta} =$ [2]

7. $\frac{\partial^2 z}{\partial r \partial \theta} =$ [2]

For questions 8-11 consider

$$z = x^2 + xy + y^2 - 6x + 2$$

8. Consider the statements: [2]

(i) $\frac{\partial z}{\partial x} = 2x + y - 6$ (ii) $\frac{\partial z}{\partial y} = x + 2y - 6$

Which is/are correct/incorrect?

9. Consider the statements: [2]

(i) $\frac{\partial^2 z}{\partial x^2} = 1$ (ii) $\frac{\partial^2 z}{\partial x \partial y} = 1$

Which is/are correct/incorrect?

10. A stationary point of z is [3]

11. The nature of the stationary point in question 10 is: [2]

12. $\int \frac{1}{\sqrt{x^2 - 4x}}\, dx =$ [3]

For questions 13-14 consider

$$\frac{-2t+6}{(t+4)(t^2-5)}$$

13. In the partial fraction decomposition of $\frac{-2t+6}{(t+4)(t^2-5)}$, the numerator of the fraction containing $t+4$ is [3]

14. In the partial fraction decomposition of $\frac{-2t+6}{(t+4)(t^2-5)}$, the numerator of the fraction containing t^2-5 is [3]

For questions 15-16 consider

$$\int t\ln t\,dt$$

15. If we have to make use of the D-I method, then the most efficient choice is [2]

(i) $D=t$ (ii) $I=\ln t$

Which is/are correct/incorrect?

16. $\int t\ln t\,dt=$ [2]

17. To calculate $\int \sin^5 2t\,dt$ we will make use of the following identity [2]

For questions 18-19 consider the differential equation

$$\frac{dy}{dt}=2e^{3t-2y};\quad y(0)=0$$

18. The general solution is [3]

19. The unique solution is [2]

For questions 20-21 consider the following matrix:

$$A=\begin{pmatrix}4 & -7 & 6\\ -2 & 4 & 0\\ 5 & 7 & -4\end{pmatrix}$$

20. The determinant of A is [3]

21. If $A^{-1}=\begin{pmatrix}a & b & c\\ d & e & f\\ g & h & i\end{pmatrix}$, then the value of e is [3]

FORMULAE SHEETS

TRIGONOMETRY			
1.	$\sin^2 A + \cos^2 A = 1$	**15.**	$\sinh^2 A - \cosh^2 A = -1$
2.	$1 + \tan^2 A = \sec^2 A$	**16.**	$1 - \tanh^2 A = \operatorname{sech}^2 A$
3.	$1 + \cot^2 A = \operatorname{cosec}^2 A$	**17.**	$\coth^2 A - 1 = \operatorname{cosech}^2 A$
4.	$\sin 2A = 2\sin A\cos A$	**18.**	$\sinh 2A = 2\sinh A\cosh A$
5.	$\cos 2A = \cos^2 A - \sin^2 A$	**19.**	$\cosh 2A = \cosh^2 A + \sinh^2 A$
6.	$\cos^2 A = \frac{1}{2}[1 + \cos 2A]$	**20.**	$\cosh^2 A = \frac{1}{2}[1 + \cosh 2A]$
7.	$\sin^2 A = \frac{1}{2}[1 - \cos 2A]$	**21.**	$\sinh^2 A = \frac{1}{2}[-1 + \cosh 2A]$
8.	$\sin(-A) = -\sin A$	**22.**	$\sinh(-A) = -\sinh A$
9.	$\cos(-A) = \cos A$	**23.**	$\cosh(-A) = \cosh A$
10.	$\tan(-A) = -\tan A$	**24.**	$\tanh(-A) = -\tanh A$
11.	$\sin A\cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$	**25.**	$\sinh A = \frac{e^A - e^{-A}}{2}$; $\cosh A = \frac{e^A + e^{-A}}{2}$
12.	$\sin A\sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$	**26.**	$\sin A = \frac{e^{jA} - e^{-jA}}{2j}$; $\cos A = \frac{e^{jA} + e^{-jA}}{2}$
13.	$\cos A\cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$	**27.**	$e^{jA} = \cos A + j\sin A$; $e^{-jA} = \cos A - j\sin A$
14.	$\operatorname{sinc} A = \frac{\sin A}{A}$		

PARTIAL FRACTIONS AND COMPLETION OF THE SQUARE	
28.	$\frac{f}{(ax + b)(cx + d)} = \frac{A}{ax + b} + \frac{B}{cx + d}$
29.	$\frac{f}{(ax + b)^n} = \frac{A}{ax + b} + \frac{B}{(ax + b)^2} + \cdots + \frac{N}{(ax + b)^n}$
30.	$\frac{f}{(ax^2 + bx + c)(dx^2 + ex + f)} = \frac{Ax + B}{ax^2 + bx + c} + \frac{Cx + D}{dx^2 + ex + f}$
31.	$\frac{f}{(ax^2 + bx + c)^n} = \frac{Ax + B}{ax^2 + bx + c} + \frac{Cx + D}{(ax^2 + bx + c)^2} + \cdots + \frac{Mx + N}{(ax^2 + bx + c)^n}$
32.	$ax^2 + bx + c = a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right]$

INTEGRATION

33. $\int k\,du = ku + c$

34. $\int u^n du = \frac{u^{n+1}}{n+1} + c, \quad (n \neq -1)$

35. $\int \frac{du}{u} = \ln|u| + c$

36. $\int e^u du = e^u + c$

37. $\int a^u du = \frac{a^u}{\ln a} + c$

38. $\int \sin u\,du = -\cos u + c$

39. $\int \cos u\,du = \sin u + c$

40. $\int \tan u\,du = \ln|\sec u| + c$

41. $\int \cot u\,du = \ln|\sin u| + c$

42. $\int \sec u\,du = \ln|\sec u + \tan u| + c$

43. $\int \sec^2 u\,du = \tan u + c$

44. $\int \sec u \tan u\,du = \sec u + c$

45. $\int \operatorname{cosec} u\,du = \ln|\operatorname{cosec} u - \cot u| + c$

46. $\int \operatorname{cosec}^2 u\,du = -\cot u + c$

47. $\int \operatorname{cosec} u \cot u\,du = -\operatorname{cosec} u + c$

48. $\int \sinh u\,du = \cosh u + c$

49. $\int \cosh u\,du = \sinh u + c$

50. $\int \tanh u\,du = \ln[\cosh u] + c$

51. $\int \coth u\,du = \ln[\sinh u] + c$

52. $\int \operatorname{sech} u\,du = \tan^{-1}[\sinh u] + c$

53. $\int \operatorname{sech}^2 u\,du = \tanh u + c$

54. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + c$

55. $\int \operatorname{cosech} u\,du = \ln\left[\tanh\frac{u}{2}\right] + c$

56. $\int \operatorname{cosech}^2 u\,du = -\coth u + c$

57. $\int \operatorname{cosech} u \coth u\,du = -\operatorname{cosech} u + c$

58. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + c$

59. $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\frac{u}{a} + c$

60. $\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}\frac{u}{a} + c$

61. $\int \frac{du}{a^2 - u^2} = \frac{1}{a}\tanh^{-1}\frac{u}{a} + c$

62. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$

NUMERICAL ANALYSIS

63. $\int_a^b y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n\right]$

64. $\int_a^b y\,dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + y_5\ldots) + 2(y_2 + y_4 + y_6 + \ldots) + y_n]$

65. $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$

66. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$