Update January 18, 2021: The grading scheme for Q1 has changed to be more generous.

MAT136H5 S - WINTER 2021 - WRITTEN ASSIGNMENT 1

Submission

- **You must submit your completed Written Assignment on Crowdmark by 6:00pm (EST) Friday January 29, 2021**. You will be emailed a link from Crowdmark with information on how to submit your solutions.
- Late assignments (even by a couple seconds) will not be accepted.
- Consider submitting your assignment well before the deadline.
- You do not need to print out this assignment; you may submit clear pictures/scans of your work on lined paper, or screenshots of your work.
- You do not need to submit the cover page, or the grading scheme.
- You must correctly orient/rotate and order your submission.
- If you require additional space, please insert extra pages.

Additional Instructions

You must justify and support your solution to each question. You should use full sentences.

Academic Integrity

You are encouraged to work with your fellow students while working on questions from the written assignments. However, the writing of your assignment must be done without any assistance whatsoever. Do not post partial or complete solutions to Piazza.

I affirm that this assignment represents entirely my own efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- This is the final version of my assignment and not a draft.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters*.

By submitting solutions for grading I agree that the statements above are true. If I do not agree with the statements above, I will not submit my assignment and will consult the course coordinator (Mike Pawliuk) immediately.

Need help?

This problem set is designed to make you think, and it contains problems you've never seen before. We expect you'll need to come back to this assignment multiple times and try different approaches; we don't expect you to solve everything in one sitting. It's normal to get stuck! Every time you get stuck that means you're about to learn something when you get unstuck. Look for those moments!

If you're stuck for more than a day or two, you may want to ask for help. Here are some places to do that:

- **Ask on Piazza**. (If you want to post some of your work, please make it a private post.)
- **Office hours**. See Quercus for times and locations. There are about 20 hours a week, and you can attend the office hours of any instructor, not just the one for your LEC section.

Good luck, have fun!

Grading Scheme

This is the grading scheme that TAs will use when grading this assignment. You do not need to submit this page.

Question 1. [5 points].

- 1 point for clear justification (probably including a map with shapes on it).
- 1 point for correct computations.
- 1 point for an answer within 1 000 000 m^2 of the true area.
- 1 point for an answer within 100 000 m^2 of the true area.
- 1 point for an answer given as a complete sentence in the correct units (m^2).

Question 2 [5 points].

- 2 points for finding the correct fraction of the famous constant. (Only award 1 point if it is the correct constant, but an incorrect fraction.)
- 2 points for clear justification.
- 1 point for a correct answer given as a complete sentence.

Question 3 [5 points].

- Part 1: 0.5 points for justifying each inequality. The justification should point out what shapes are being used.
- Part 2: 2 points total. 1 point for an appropriate diagram. 1 point for justifying both inequalities (0.5 points each).
- Part 3: 2 points total. 1 point for setting up a correct integral with respect to y, and 1 point for justifying the integral.

Question 4 [5 points].

- Part 1: 1 point for any reasonable justification.
- Part 2: 4 points total. 1 point for choosing an appropriate substitution. 1 point for correctly making the substitution. 1 point for algebra. 1 point for a correct final answer.

Question 1. Approximate the total area of Centre Island (also known as the Island of Hiawatha, or Menecing) in Toronto.

- You may use any approximation method you want.
- Only include Centre Island, and do not include any other islands connected by bridges.
- Give your answer in m^2.
- You may use the map `island_map.jpg` [1] posted on Quercus. It contains a scale.

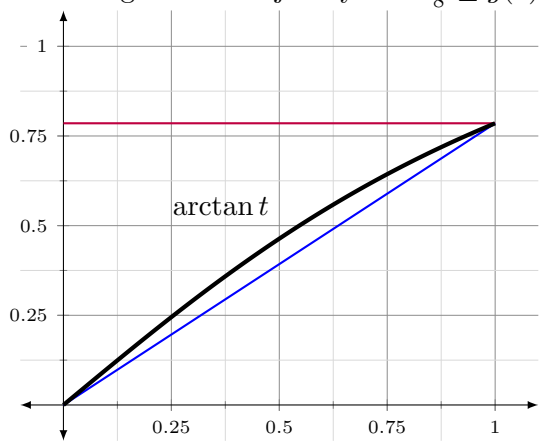
[1] The map is "©OpenStreetMap contributors" and is used with permission from OpenStreetMap `https://www.openstreetmap.org/copyright`.

Question 2. This limit evaluates to a fraction of a famous constant. Find it!

$$C = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\sqrt{1 - \frac{i^2}{n^2}}$$

Question 3. In this question you will be exploring the function $g(x) = \int_0^x \arctan t \, dt$.

(1) Use the image below to justify that $\frac{\pi}{8} \leq g(1) \leq \frac{\pi}{4}$.



(2) Let $x > 0$. Create a similar image for $\dfrac{x \arctan x}{2} \leq g(x) \leq x \arctan x$, and use it to justify these inequalities.

(3) Use your picture from (2), $\int \tan t \, dt = \ln(\sec t) + C$, and some algebra, to justify why

$$\int_0^x \arctan t \, dt = x \arctan x - \frac{1}{2} \ln(1 + x^2).$$

(Hint: Set up an integral with respect to y in your picture and identify $x \arctan x$ as an area.)

Question 4. The goal of this question is to evaluate

$$I = \int \frac{x^8}{\sqrt{x^3 - 1}}\, dx.$$

(1) Read all the examples in Section 5.5 of the textbook. Which of the examples is I most similar to? Explain why (in your own words).

(2) Evaluate the integral I.