INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Department of Chemical Engineering

CH5230 System Identification

Assignment 1

Due: Wednesday, February 24, 2021

Exercises

1. (a) Verify that for the 3-parameter model example in class, the input $u[k] = \cos^2(\omega_0 k)$ results in loss of identifiability. In general, using linear algebra principles, derive the condition that the input $u[k]$ should satisfy.
   (b) Examine the identifiability of a first-order LTI system $y[k]+a_1 y[k-1] = u[k]+a_1 u[k-1]$ for two different conditions (i) non-zero $u[k]$ and zero $y[0]$ (forced response) and (ii) zero $u[k]$ but non-zero $y[0]$ (free response).

2. The impulse response of a d.t. system is given by $g[k] = 2(0.6)^{k-1} + (0.4)^{k-1},\ k \geq 1.$

   a) Plot the impulse and step responses of the system. Is the system causal and stable?
   b) Arrive at the frequency response function $G(e^{j\omega})$ of the system. Plot the magnitude and phase response of the system.
   c) What is an appropriate FIR approximation of the system?

3. A deterministic d.t. LTI system is governed by the following difference equation:

$$y[k] - 1.3y[k-1] + 0.24y[k-2] = 3u[k-1] \tag{1}$$

   a) What is the delay, order and gain of the system?
   b) Determine the IR of the system by solving (1). Assume a *relaxed* system.
   c) Express (1) as two first-order difference equations (introduce artificial variables) change?

4. With reference to the liquid level case study[1], generate a fresh flow-level response data by changing the measurement noise from *white* to *filtered* noise by placing a *discrete-time* filter block in the disturbance channel path, in the SIMULINK block diagram. Use the filter coefficients

   `num = 1, den =` $[1 \ \ -0.9]$

   and set the SNR to 10. For this purpose, switch off the noise, simulate and compute the variance of the true response. Subsequently, turn the white-block noise on and adjust the variance parameter in the block such that the desired SNR is achieved.

   (a) Using the obtained data, determine the best OE model (based on the two tests of correlation between inputs and residuals, and errors in parameter estimates) still manages to explain the deterministic process correctly.
   (b) How does the ARX model (of a suitably high-order) fare in the above situation, i.e., data generated with colored noise?

[1] Use the SIMULINK model posted on the course site.