MFIT5009: Optimization in Fintech	**(Due: 27/02/2021)**

Homework #1

Instructor: Daniel Palomar — *Name:* Student name(s), *Netid:* NetId(s)

Course Policy: Read all the instructions below carefully before you start working on the assignment and before you make a submission.

- Please typeset your submissions in LaTeX, RMarkdown or Jupyter notebook. Please include your name and student ID with submission. Submit your homework in the form of pdf or html via Canvas.
- Assignments are due by 11:59 pm of February 27th.
- No cheating will be tolerated, so make sure you complete the homework on your own.
- Only R and Python are allowed for the implementation. You should also show the code for how to generate the function and remember to set seed if needed so that your code can be replicated.

Problem 1: Practice with Solvers (45 points)

Please read this website carefully before you start solving the problems.

(a) Solve the following optimization problem:

$$\begin{array}{ll} \underset{\boldsymbol{\Sigma}}{\text{maximize}} & \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} \\ \text{subject to} & L_{jk} \leq \Sigma_{jk} \leq U_{jk}, \quad j,k = 1,\ldots,n. \end{array}$$

Here, we consider

$$\mathbf{w} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.5 \end{pmatrix}, \quad \mathbf{L} = \begin{pmatrix} 0.2 & 0.1 & 0.1 \\ -0.5 & 0.1 & 0.2 \\ -0.3 & 0.1 & 0.1 \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} 0.2 & 0.7 & 0.4 \\ 0.4 & 0.1 & 0.5 \\ -0.1 & 0.5 & 0.7 \end{pmatrix}.$$

Please show the optimal value of $\boldsymbol{\Sigma}$ and all the code you use to solve the problem.

(b) Solve the following optimization problem:

$$\begin{array}{ll} \underset{\mathbf{X}\in S_+^n}{\text{minimize}} & \lambda_{\max}(\mathbf{X}) - \lambda_{\min}(\mathbf{X}) \\ \text{subject to} & \operatorname{trace}(\mathbf{A}\mathbf{X}) = 1. \end{array}$$

Here, we consider

$$\mathbf{A} = \begin{pmatrix} 91 & 83 & 85 & 54 \\ 83 & 102 & 84 & 56 \\ 85 & 84 & 150 & 72 \\ 54 & 56 & 72 & 52 \end{pmatrix}.$$

Please show the optimal value of $\mathbf{X}$ and all the code you use to solve the problem.

(c) Solve the unconstrained optimization problem with piecewise-linear objective function

$$\underset{x\in\mathbb{R}}{\text{minimize}} \quad \max_i\{a_i x + b_i\},$$

where $\mathbf{a} = (a_1, a_2, a_3, a_4, a_5) = (-2, -4, -8, 1, 3)$ and $\mathbf{b} = (b_1, b_2, b_3, b_4, b_5) = (-1, -6, -1, -1, -12)$. Compare the results with the solution of following optimization problem:

$$\begin{array}{ll} \underset{x\in\mathbb{R}}{\text{minimize}} & t \\ \text{subject to} & t \geq a_i x + b_i, \; i = 1,\ldots,5. \end{array}$$

Please show the optimal value of x for both problems and all the code you use to solve the problem.

Problem 2: Practice with key packages for finance (25 points)

In this problem we need to import SP500 index from Jan 3, 2005 to Dec 31, 2016.
(a) Convert daily data to monthly data by the index to the first day of the month. Then plot the corresponding monthly prices using 'ggplot2'.

(b) Compute the 10-day simple moving average and 10-day exponential moving average for daily closing prices, draw them on the same figure.

Problem 3: Elastic net regularization (30 points)

In statistics, the Elastic net uses a regularization that linearly combines the L1 and L2 penalties. The estimates from the elastic net method can be denoted as

$$\hat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \left(f(\boldsymbol{\beta}) + \lambda_1 \|\boldsymbol{\beta}\|_1 + \lambda_2 \|\boldsymbol{\beta}\|_2^2 \right),$$

where $f(\boldsymbol{\beta})$ is the loss function without regularization subject to the optimization variable $\boldsymbol{\beta}$ and λ_1, λ_2 are the coefficients to control the magnitude of penalty. Now we consider a linear model with

$$f(\boldsymbol{\beta}) = \|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|_2^2$$

where $\mathbf{y} \in \mathbb{R}^n$, $\mathbf{X} \in \mathbb{R}^{n\times p}$, and $\boldsymbol{\beta} \in \mathbb{R}^p$. Please find the data of X and y with $n = 500$ and $p = 200$ via Canvas, solve the problem with different values of (λ_1, λ_2), then fill in the table by computing

$$\sum_{i=1}^{p} \mathbb{I}\left(|\beta_i^\star| \le \epsilon\right),$$

where $\boldsymbol{\beta}^\star$ is the corresponding optimal solution, ϵ is set to be 10^{-3}, and the indicator function is defined as

$$\mathbb{I}(|\beta_i^\star| \le \epsilon) = \begin{cases} 1 & \text{if } |\beta_i^\star| \le \epsilon, \\ 0 & \text{otherwise.} \end{cases}$$

	$\lambda_1 = 10^0$	$\lambda_1 = 10^2$	$\lambda_1 = 10^4$	$\lambda_1 = 10^6$
$\lambda_2 = 10^0$				
$\lambda_2 = 10^2$				
$\lambda_2 = 10^4$				
$\lambda_1 = 10^6$				